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OSWALD VEBLEN

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George D. Birkhoff

GEORGE DAVID BIRKHOFF

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BY OSWALD VEBLEN

GEORGE DAVID BIRKHOFF was born at Overisel, Michigan, on the twenty-first of March, 1884. His ancestry was Dutch on both sides. His father, David Birkhoff, came from Holland in 1870, and during George David's growing years was a physician in Chicago. Birkhoff studied at the Lewis Institute, Chicago, from 1896 to 1902, and at the University of Chicago for a year. After this he went to Harvard, where he received the Bachelor's degree in 1905.

Beginning in the year 1900 there appeared in the problem department of the *American Mathematical Monthly*, edited by B. F. Finkel, a series of notes, solutions, and problems by H. S. Vandiver, of Bala, Pennsylvania. In 1901 Birkhoff, who had doubtless found the monthly in the old John Crerar Library, began exchanging letters about various questions in the theory of numbers with Vandiver, who was then nineteen years old. This correspondence resulted in the publication in 1904 of their joint paper in the *Annals of Mathematics* "On the integral devisors of a^n-b^n." So far as I know this was Birkhoff's only publication in the theory of numbers,

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but Vandiver has told me that Birkhoff was in possession in those days of at least one number-theoretical theorem which is now counted among the notable contributions of a distinguished mathematician in another part of the world. In later life Birkhoff often showed an interest in number theory, but seems never to have taken the deep plunge which would have been necessary in order to bring up new results of the sort that would have satisfied him. It was not until his Princeton period that he met Vandiver personally.

During his undergraduate years he also made a definite beginning in analysis, as is proved by the fact that he read a paper entitled "A general remainder theorem" before the American Mathematical Society in New York in February 1904 (*Amer. Math. Soc. Bulletin* 10: 280). This was the basis of a paper entitled "General mean value and remainder theorems with applications to mechanical differentiation and quadrature," published in the *Transactions of the American Mathematical Society*, volume 7 (1906).

Birkhoff returned to the University of Chicago in the fall of 1905 and received his Ph.D. *summa cum laude* in 1907 at the age of twenty-three. This is not an unusual age for a European doctorate but, unfortunately for the New World, it is an exceptionally early one in the United States. Birkhoff's student period had been divided between the only two great mathematical centers which existed in America at that time. From Osgood and Bôcher he obtained a thorough introduction to the classical methods of analysis, and from E. H. Moore who was then at the outset of his adventure in "General Analysis," a grasp of the abstract modernistic ideas which have characterized so much of mathematics during the last four decades. Birkhoff reacted rather strongly against the latter and in favor of the former. His view was that while one should understand the analogies between the linear problems of analysis and those of classical geometry and algebra, his attention should be concentrated on strategically important specific problems of the classical type.

His doctoral dissertation on asymptotic problems of ordinary linear differential equations does in fact continue the tradition to which Bôcher belonged. But it also uses the powerful methods of the Fredholm theory of integral equations and the broad general ideas which E. H. Moore was trying to exploit. It initiated a series of studies by which he left his mark on most of the principal branches of the theory of linear differential equations: regular and irregular singular points, expansion and boundary value problems, separation theorems, and his generalization of the Riemann problem. With these researches it seems reasonable to group his work on matrices of analytic functions and his remarkable contributions to the theory of linear difference equations, as constituting one of the three principal periods of Birkhoff's scientific activity. In time, this period overlaps his whole career, but his most intense effort in these fields belongs to his earlier years.

After receiving his doctorate in 1907, Birkhoff spent two years in Madison as an instructor in the University of Wisconsin. Here he learned more analysis from E. B. Van Vleck, and in particular had his attention directed toward linear difference equations. This period also includes his marriage in 1908 to Miss Margaret Elizabeth Graftus, a union of mutual devotion and helpfulness which lasted throughout the rest of his life. There were three children, Barbara (Mrs. Robert Treat Paine, Jr.), Garrett, and Rodney. Garret has already gained distinction as a mathematician of quite different tendencies from those of his father.

In 1909 he came to Princeton University as a preceptor and was promoted to a professorship on the occasion of a call to Harvard in 1911. At Princeton during this period a third significant current of American mathematical thought, a geometrical one, was gathering force. Birkhoff shared in the exploratory studies then being made of analysis situs, as it was called before being formalized into "topology," and saw their close relation to the class of dynamical problems which were at this time taking definite form in his mind. Incidentally, he had more than one try at the four-color map problem, to solve which remained throughout life one of his dearest aspirations.

In 1912 Birkhoff reconsidered the question of returning to Harvard, and accepted an assistant professorship in that university, in which rank he remained for another seven years. He thus returned to the most stable academic environment then available in the country, and settled into a long period of creative work undisturbed by necessity, common in American universities of this epoch, to build an environment in which scientific work can bear fruit. The final transition to Harvard was recognized by Birkhoff himself and his most intimate friends as marking the end of the formative period of his career. I remember in particular a delightful letter which he received from E. H. Moore, ending with the words written out in bold characters: AVE ATQUE VALE.

As remarked by Marston Morse, however, "Poincaré was Birkhoff's true teacher." I remember well how frequently, in the walks we used to take together during his sojourn in Princeton, Birkhoff used to refer to his reading in Poincaré's *Les Méthodes Nouvelles de la Méchanique Céleste*, and I know that he was intensively studying all of Poincaré's work on dynamics. In a very literal sense Birkhoff took up the leadership in this field at the point where Poincaré laid it down.

Poincaré died in 1912 and his last paper reached Princeton in the summer of that year. In it Poincaré showed that the existence of periodic solutions of the restricted problem of three bodies can be deduced from a very simplesounding geometric theorem. But he had not been able to prove the theorem except in special cases, and he felt that at his age (he was only fifty-eight when he died) he could not be sure of being able to return to it again, as he should have liked to do, after letting his ideas lie fallow for a while. Before the year was over Birkhoff had given a simple but profound proof of “Poincaré’s Geometric Theorem.” The publication of this proof in the *Transactions of the American Mathematical Society* for January 1913 brought immediate and worldwide fame to its author, an acclaim which, for once, was justified by subsequent events.

His researches in dynamics constitute the middle period of Birkhoff’s scientific career, that of maturity and greatest power. Their chief characteristics can be seen already in his first publication, “Quelques théorèms sur le mouvement des systèmes dynamiques,” which appeared in the *Bulletin de la Société Mathématique de France* in 1912. In this paper after a careful examination of the properties of stable motion, Birkhoff introduced his concept of “recurrent motion” which has played a role alongside the classical concept of periodic motion in all further discussions of the descriptive properties of dynamical trajectories. It is, for example, the starting point of the “symbolic dynamics” of Morse and Hedlund. While Poincaré had made good use of topology in the theory of dynamical systems, it was Birkhoff’s merit to have powerfully supplemented this by the use of the Lebesgue measure theory. In the unfolding of the geometric picture of the general case in dynamics, one of the significant stages was the introduction of the concept of “metric transitivity” which appeared for the first time in his joint paper with Paul Smith on “Structure analysis of surface transformations” in *Liouville’s Journal* (1928) where it was applied to two-dimensional problems of a class more general than those of dynamics. This line of thought reached its climax in the winter of 1931-32 when under the stimulus of closely related discoveries by Koopman and von Neumann he succeeded in proving his justly famous "ergodic theorem." Birkhoff's ergodic theorem, though it does not completely solve the basic problem of statistical mechanics at which it is aimed, has reduced that problem to a definite question about metric transitivity, and is also a milestone in the progress of measure theory. Birkhoff's proof, which, characteristically, used the rough and ready tools picked up along the path which led him to it, has been replaced by simpler and more sophisticated methods, and there has grown up a rather extensive literature of "ergodic theory."

Most of Birhoff's publications in dynamics are devoted to dynamical systems of two or three degrees of freedom. Here he enjoyed Poincaré's concept of a "surface of section" and the transformations in it determined by a family of dynamical trajectories. Poincaré's geometric theorem is a case in point. He also carried the use of the representation of trajectories by means of geodesics on surfaces considerably beyond the stage reached by Poincaré and Hadamard. His "minimax principle" was the starting point of Morse's "Analysis in the Large" which has done so much to make topology effective in analysis.

Although Birkhoff's most notable successes were in the geometrical aspect of dynamics, he did not neglect, nor was he deficient in power over the analytic formalism. He achieved as good a view of the whole field of theoretical dynamics as did anyone in his time. For more authoritative accounts and evaluations of Birkhoff's work both in this field and in what I have called his first period, I should like to refer the reader to the notices by E. T. Whitaker in the *Journal of the London Mathematical Society*, volume 20 (1945), and by Marston Morse in the *Bulletin of the American Mathematical Society*, volume 52 (1946). In addition, there are many interesting comments on his own work and revelations of his point of view toward that of his contemporaries, in Birkhoff's address on "Fifty years of American mathematics" which was published in 1938 in a volume celebrating the semicentennial of the American Mathematical Society.

The third phase of Birkhoff's scientific career was that in which he sought to extend mathematical methods into other fields of thought,—physics, aesthetics, and even ethics. He was already speculating on the possibility of a mathematical theory of music, and indeed of art in general, while he was in Princeton. But he did not give these ideas to the world until 1928 when he delivered one of the principal addresses of an international mathematical congress under the title, "Quelques eléments mathématiques de l'art," in the Salon del Cinquecento of the Palazzo Vecchio at Florence. Later on, after much reflection and a trip around the world, he published his book *Aesthetic Measure,* in 1933. In 1942, "A mathematical approach to ethics" appeared in the *Rice Institute Pamphlets* (vol. 28). These studies, though Birkhoff took them quite seriously, seem to me to be definitely less likely than his purely mathematical work to survive.

Something similar, I think, must be said about his efforts in physics. Like Goethe and Hilbert, he always remained an outsider. It may have been that the very strength of his faith in mathematical insight prevented him from properly appreciating the insight of the physicists. His active interest in physics seems to have begun with a course in relativity which he gave in the winter of 1921-1922 and it continued increasingly up to the time of his death when he was engaged in exploiting a gravitational theory of his own. As his contribution to physics there remain some unquestionable improvements in mathematical technique, some criticisms of present tendencies, and a physical theory which can survive only if it passes the tests both of experiment and assimilability into the growing body of science.

Among the unconscious revelations of the address on "Fifty years of American mathematics," one of the most vivid is that of the depth and sincerity of Birkhoff's devotion to the cause of mathematics, and particularly of "American mathematics." This, along with his devotion to Harvard, was always a primary motive. It may be added that a sort of religious devotion to American mathematics as a "cause" was a characteristic of a good many of his predecessors and contemporaries. It undoubtedly helped the growth of the science during this period. By now, mathematics is perhaps strong enough in the United States to be less nationalistic. The American mathematical community has at least been healthy enough to absorb a pretty substantial number of European mathematicians without serious indigestion.

Birkhoff was always on the lookout for talent among the young mathematical aspirants who came to Harvard. I recently looked over some of his letters and found them full of comments on the young men for whom he had hopes. Some of the names I had forgotten, but many of the comments are still enjoyable. His capacity for intelligent study of the qualifications and needs of younger mathematicians was used for the benefit of science on a much wider stage during the years (1925 to 1937) that Birkhoff, Bliss, and I were the mathematical members of the National Research Fellowship Board. I am sure that Bliss will agree with me about Birkhoff's remarkable capacity for picking "the good ones" and guessing what they needed. While Birkhoff was subject to as many prejudices as most of us, he kept always what most of us lose as we grow older, the power to see people and events simply and naively rather than with reference to current opinion.

Birkhoff unhesitatingly accepted the public responsibilities that came his way. He served as Dean of the Faculty of Arts and Sciences at Harvard from 1937 to 1939. He carried his share of military research work during both World Wars. He traveled extensively and accepted a large number of invitations to lecture, both those of an honorific sort and those that simply afforded an opportunity to extend mathematical culture into new areas. He did much of the unrewarded administrative work of the American Mathematical Society. For example, he served on the committee which, after a lively debate, decided to undertake the publication of *Mathematical Reviews.* After the main issues had been decided against his judgment, he cooperated loyally and actively in the working out of details.

It is pleasant to record that Birkhoff received nearly all the distinctions, such as honorary degrees and elections to societies and academies, that can come to a mathematician, and received many of them at an unusually early age. He became a member of the American Philosophical Society in 1921 and was a frequent attendant at its meetings.

During the last few years of his life Birkhoff knew that his heart was no longer as strong as it had been, but he never slackened up his scientific and other work. He died in his sleep on November 12, 1944.

SELECTED BIBLIOGRAPHY

1909

Singular points of ordinary linear differential equations. *Trans. Am. Math. Soc.* 10:436-70.

1911

On the solutions of ordinary linear homogeneous differential equations of the third order. *Ann. Math.* 12:103-27.

General theory of linear difference equations. *Trans. Am. Math. Soc.* 12:243-84.

1912

A determinant formula for the number of ways of coloring a map. *Ann. Math.* 14:42-46.

1913

The reducibility of maps. *Am. J. Math.* 35:115-28.

The generalized Riemann problem for linear differential equations and the allied problems for linear difference and q-difference equations. *Proc. Am. Acad. Arts Sci.* 49:521-68.

Proof of Poincare's geometric theorem. *Trans. Am. Math. Soc.* 14:14-22.

1915

The restricted problem of three bodies. *Rend. Circ. Mat. Palermo* 39:265-334.

1917

Dynamical systems with two degrees of freedom. *Trans. Am. Math. Soc.* 18:199-300.

1920

Surface transformations and their dynamical applications. *Acta Math.* 43:1-119.

1922

With O. D. Kellogg. Invariant points in function space. *Trans. Am. Math. Soc.* 23:96-115.

1923

With R. E. Langer. The boundary problems and developments associated with a system of ordinary linear differential equations of first order. *Proc. Am. Acad. Arts Sci.* 58:49-128.

With R. E. Langer. *Relativity and Modern Physics.* Cambridge, Mass.: Harvard University Press.

1927

Dynamical systems. *Am. Math. Soc. Colloq. Publ,* vol. 9.

A mathematical critique of some physical theories. *Bull. Am. Math. Soc.* 33:165-81.

1930

On the number of ways of coloring a map. *Proc. Edinb. Math. Soc.* 2:83-91.

1931

Proof of a recurrence theorem for strongly transitive systems; proof of the ergodic theorem. *Proc. Natl. Acad. Sci. U. S. A.* 17:650-60.

1932

With B. O. Koopman. Recent contributions to the ergodic theory. *Proc. Natl. Acad. Sci. U. S. A.* 18:279-82.

1933

With D. C. Lewis, Jr. On the periodic motions near a given periodic motion of a dynamical system. *Ann. Mat. Pura Appl.* 12:117-33.

With W. J. Trjitzinsky. Analytic theory of singular difference equations. *Acta Math.* 60:1-89.

1934

On the polynomial expressions of the number of ways of coloring a map. *Ann. Sc. Norm. Super. Pisa* 3:1-19.

1935

Sur le probleme restreint des trois corps (premier memoire). *Ann. Sc. Norm. Super. Pisa* 4:267-306.

Nouvelles recherches sur les systemes dynamiques. *Mem. Pont. Acad. Sci. Novi Lycaei* 1:85-216.

With M. R. Hestenes. Natural isoperimetric conditions in the calculus of variations. *Duke Math. J.* 1:198-286.

1936

Sur le probleme restreint des trois corps (second memoire). *Ann. Sc. Norm. Super. Pisa* 5:1-42.