Solomon Lefschetz

Russian born U.S. mathematician **Solomon Lefschetz** (September 3, 1884 – October 5, 1972) did fundamental work in algebraic topology, with applications to algebraic geometry and differential equations. He possessed great geometrical intuition and insight that led him to many original and creative results. In *Algebraic Geometry and Topology*, W.V.D. Hodge wrote: "Our greatest debt to Lefschetz lies in the fact that he showed us that a study of topology was essential for all algebraic geometers." As professor at Princeton University (1924 – 1953), Lefschetz exercised tremendous influence over American mathematics. Through his efforts as editor, the *Annals of Mathematics* (1928 – 1958) became arguably the world's most respected mathematical journal.

Lefschetz was born in Moscow to Jewish parents, who were Turkish citizens. Shortly after his birth the family resettled in Paris where he and his brothers and a sister received their schooling. He recalled that he had been "mathematics mad" since his first contact with geometry at age thirteen. Despite this, he studied mechanical engineering at the École Centrale in Paris, graduating in 1905 as *ingénieur des arts et manufactures*, the third youngest in a class of 220. As foreigners had little opportunity of securing academic posts in France, Lefschetz moved to the United States. At the age of 23, while working as a member of the engineering staff of the Westinghouse Electric and Manufacturing Company in Pittsburgh, he tragically lost both his hands and forearms due to a transformer explosion. He was fitted with artificial hands worn inside a pair of shiny gloves. Later when he taught, a student would push a piece of chalk into his hand at the beginning of class and remove it at the end.

Embittered and depressed, Lefschetz decided his future did not lie in engineering but in mathematics. He prepared himself by studying the work of Émile Picard and Paul Appel, professors at the École Central, each of whom had written a three-volume treatise on mathematics. In 1910 he enrolled in a doctoral program at Clark University in Worcester, Massachusetts. After only one year he obtained his PhD, for a thesis *On the Existence of Loci with Given Singularities*, supervised by William E. Story. The project was to find the largest number of cusps that a plane curve of given degree might possess. While at Clark, Lefschetz met his wife Alice Berg Hayes who received a master's degree in mathematics from the university in 1911. They married in 1913 a year after Lefschetz became an American citizen. She was always a great influence in his life and helped him overcome his handicap. They had no children.

From 1911 to 1913, Lefschetz had a position at the University of Nebraska, and in 1913 he joined the faculty of the University of Kansas. His research became known throughout the United States and Europe. In 1919 his innovative use of topological methods earned him the Bordin Prize of the French Academy of Sciences, and in 1923 the Bôcher Prize of the American Mathematical Society. That same year Lefschetz left Kansas to accept a position at Princeton University where he remained the rest of his academic career. In 1933 he succeeded Oswald Veblen as Henry Burchard Fine research professor. Lefschetz served as chairman of the department of mathematics from 1945 until his retirement in 1953. Through his determination and energy, the department became one of the best in the world.

In *A Beautiful Mind* Sylvia Nasar gushes: "Princeton in 1948 was to mathematicians what Paris once was to painters and novelists … and ancient Athens to philosophers and playwrights." As for Lefschetz she anoints him, "the supercharged human locomotive that had pulled the Princeton department out of genteel mediocrity right to the top." This is the language of a professor of journalism, not a

mathematician. For two decades after 1935, Princeton produced more mathematics PhDs than any other American university. Lefschetz alone had thirty doctoral students. Princeton's doctoral program was unique in that there were no formal course requirements and students were encouraged to do original research from the very beginning. The program was noted for its spirit of collegial study and research among the students and faculty.

Lefschetz published eight books and more than 100 articles. In the 1920's he concentrated on topology proving his famous "fixed point theorem." With certain nice conditions, it asserts that a map f from a space to itself has a fixed point, that is, a point x for which $f(x) = x$. In the course of his work, Lefschetz invented many of the basic tools of algebraic topology. In his famous 1924 monograph *L 'analysis situs et la géométrie algébriques*, he wrote: "It was my lot to plant the harpoon of algebraic topology into the body of the whale of algebraic geometry." In 1930 Lefschetz introduced the word "Topology," from the Greek root *topos* ("place"), as a replacement for the clumsy former designation, "analysis situs." His monograph of that title brought the subject to the forefront of pure mathematics. Johann Benedict Listing used the term "topologie" in *Vorstudien zur Topologie* (1847), which he had used in his correspondence for some ten years.

During WWII, Lefschetz was a consultant with the U.S. Navy, studying guidance systems and the stability of ships. He became acquainted with the work of the Soviet mathematicians on nonlinear mechanics and control theory. He turned his attention to the geometric approach to nonlinear differential equations. Through government funding for a differential equations project, directed by Lefschetz, Princeton became a leading center for the study of ordinary differential equations. After leaving Princeton, Lefschetz convinced the Martin Company, an aircraft manufacturer, to establish a mathematics research center devoted to research on differential equations. In 1964 it was moved to Brown University and became the Lefschetz Center for Dynamical Systems. Lefschetz took to

spending his summers in Mexico where he helped build a flourishing school. For his efforts, in 1964 the Mexican government awarded him the Order of the Aztec Eagle, its highest honor for non-citizens who rendered distinguished service to Mexico. One of his fellow honorees that year was Price Philip, Duke of Edinburgh. That same year Lefschetz was awarded the National Medal of Science as head of the department, “for indomitable leadership in developing mathematics and training mathematicians, for fundamental publications in algebraic geometry and topology, and for stimulating needed research in nonlinear control processes.”

When Lefschetz arrived at Princeton he said he was often made to feel like an “invisible man.” As one of the first Jews on the faculty, proper Princeton academics pretended not to see the loud, rude, and poorly dressed outsider. According to Uwe K. Faulhaber, in an article “Intellectual Migration” written for the Simon Wiesenthal Center, “When Solomon Lefschetz was elected president of the American Mathematical Society (1935 - 1936) various leading members of the Society expressed fears that the new president would work for his own race; that this was a further sign of political interference by Jews in the United States; and that now Jewish members, Solomon Lefschetz included, would become ‘very cocky, very racial.’” Later as head of the mathematics department Lefschetz was criticized for refusing to admit many qualified Jewish students. His questionable rationale for this was that he felt no one would hire them even if they had a Princeton degree.

Lefschetz valued independent thinking and originality above all else. He had no patience with those who demanded elegance in their work. Once he was convinced that something was true he saw no need to polish the apple. Paul Halmos said of Lefschetz: “He saw mathematics not as logic but as pictures.” Although Lefschetz introduced the phrases “topology” and “algebraic topology” he dismissed point-set topology as some more “baby stuff.” Once when a student showed him a shorter proof of one of Lefschetz’s theorems, he roared: “Don’t come to me with your petty proofs. We don’t bother with that

baby stuff around here." Perhaps it was this attitude that inspired the joke about Lefschetz, namely "that he never wrote a correct proof or stated an incorrect theorem." In *Indiscrete Thoughts*, Gian-Carlo Rota wrote that Lefschetz "despised mathematicians who spent their time giving rigorous or elegant proofs for theorems which he considered obvious." Rota wrote that Lefschetz's topology textbook "hardly contains one completely correct proof."

Lefschetz could be harsh and he could be wrong. His bullying tactics could be intimidating to new graduate students, no matter how brilliant. Quoting Nasar again: "On (John Forbes) Nash's second afternoon in Princeton, Solomon Lefschetz rounded up the first-year graduate students in the West Common Room. He was there to tell them the facts of life, he said, in his heavy French accent, fixing them with a fierce gaze. And for an hour Lefschetz glared, shouted, and pounded the table with his gloved, wooden hands, delivering something between a biblical sermon and a drill sergeant's diatribe." He was infamous for badgering students and established mathematicians alike. The students good-naturedly composed a ditty about their bossy and bullying professor,

Here's to Lefschetz, Solomon L.
Irrepressible as hell
When he's at last beneath the sod
He'll then begin to heckle God.

Mathematicians enjoy tracing their mathematical lineage. My Ph.D thesis was in algebraic topology, and I often entertained myself by reciting my mathematical lineage for my students, who as captive audiences may have been less enchanted. It was, "My advisor is Jim Stasheff, his advisor was John C. Moore, whose advisor was George Whitehead, Jr., whose advisor was Norman Steenrod, whose advisor was Solomon Lefschetz and Lefschetz lives and before him there was darkness." This was a bit of

innocent hyperbole because Lefschetz is credited with originating algebraic topology as a distinct branch of mathematics. Lefschetz's dissertation at Clark University was not on algebraic topology. Although most of the development of algebraic topology occurred in the 20th century, it had a long pre-history, with its roots in the polyhedron theorem of Leonhard Euler. Other names that are associated with the early development of the mathematical field include Enrico Betti, G.F.B. Riemann, and Henri Poincaré. Simply stated, algebraic topology is the study of topological spaces using algebraic methods to a large extent. Combinatorial topology was an older name for algebraic topology, implying an emphasis on how a space was constructed from simpler ones. Now the basic method of algebraic topology is to investigate spaces via algebraic invariants, which reflect some of the topological structure of the spaces.

Quotation of the Day: "If it's just turning the crank, it's algebra; but if there is an idea present then it's topology." – Solomon Lefschetz